★ Bootstrap & Wilson Line in $\mathcal{N}=4$ SYM ★

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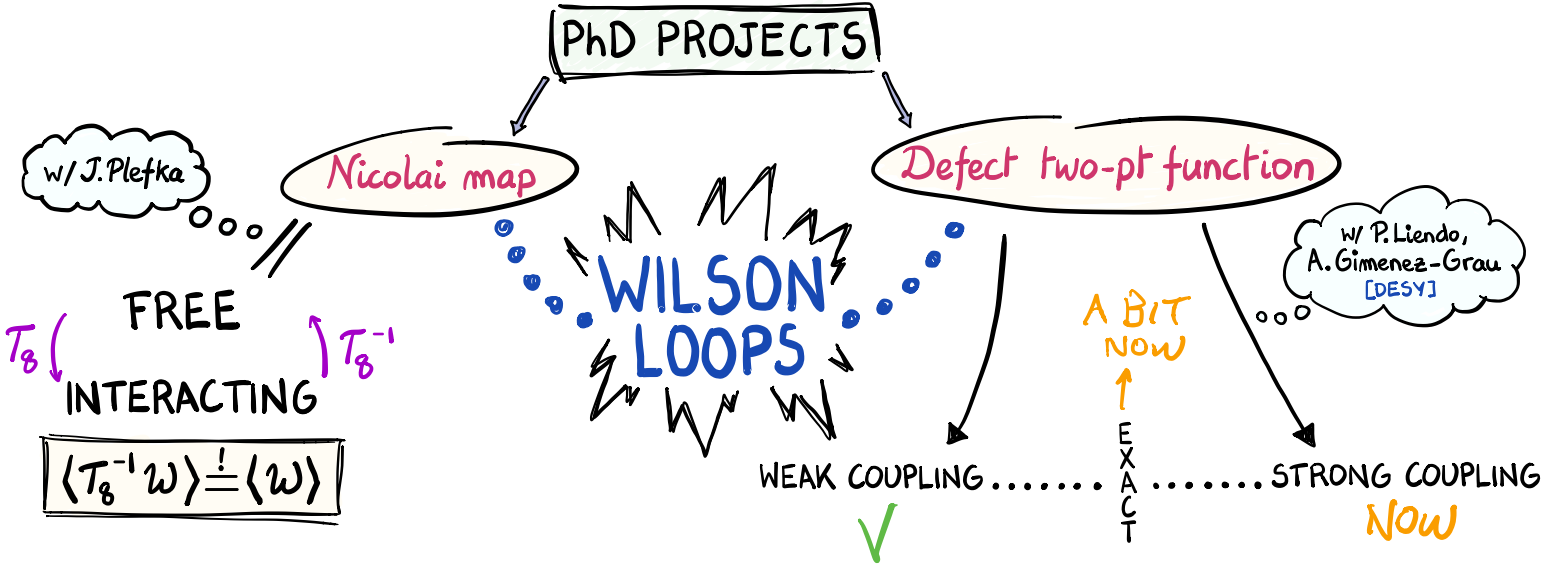
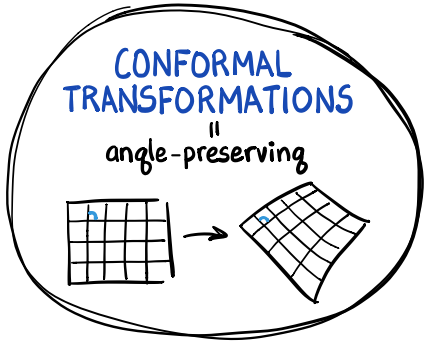
ArXiv: 2011.04678 [JB, P. Liendo, J. Plefka]

→ talk: Friday, 11 Dec at 2pm



OUTLINE:
1. Conformal symmetry & line defect
2. Bootstrap of two-pt function
3. CFT Data



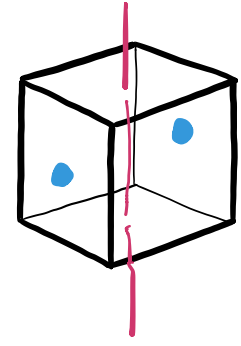
1. CONFORMAL SYMMETRY & LINE DEFECT

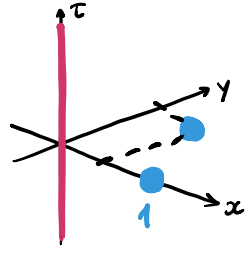
DEFECT CFT: $SO(1,5) \to SO(1,2) \times SO(3)$

1d CFT ∥ line; rotations ⊥ line

TWO-PT FUNCTION: $\langle W_\ell\, \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\rangle \sim \frac{1}{x_1^2 x_2^2}\, \mathcal{F}(z,\bar{z},\omega)$ — $SO(6)_R$

OPE:
- $\mathcal{O}_2(x_1)\mathcal{O}_2(x_2) \sim \sum_{\mathcal{X}} \lambda_{\mathcal{O}_2\mathcal{O}_2\mathcal{X}}\, \mathcal{X}(x_1)$
- $W_\ell\, \mathcal{O}_2(x) \sim \sum_{\hat{\mathcal{X}}} b_{\mathcal{O}_2\hat{\mathcal{X}}}\, \hat{\mathcal{X}}(x)$





$z = x + iy$, $\bar{z} = x - iy$

CHIRAL PRIMARIES

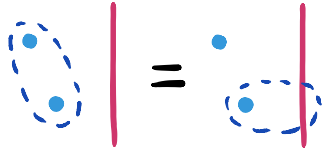
$\mathcal{O}_2 \sim \mathrm{Tr}\,\phi^i\phi^j$ ← $i,j = 1,\dots,6$

(antisymmetric traceless)

WILSON LINE

$W_\ell = \frac{1}{N}\mathrm{Tr}\,\mathcal{P}\exp\int_{-\infty}^{\infty} d\tau \left(iA_\mu(\tau)\dot{x}^\mu(\tau) + \theta_i \Phi^i(\tau)\right)$

2. BOOTSTRAP OF TWO-PT FUNCTION

CROSSING SYMMETRY: 

$$\Leftrightarrow \mathcal{F}(z,\bar{z},\omega) \sim \sum_{\chi} a_\chi \lambda_{\mathcal{O}_2\mathcal{O}_2\chi}\, \mathcal{G}_\chi(z,\bar{z},\omega) \sim \sum_{\hat{\chi}} b^2_{\mathcal{O}_2\hat{\chi}}\, \hat{\mathcal{G}}_{\hat{\chi}}(z,\bar{z},\omega)$$

WHAT DO WE NEED? → spectrum χ or $\hat{\chi}$ 🙂

→ blocks $\mathcal{G}_\chi$ or $\hat{\mathcal{G}}_{\hat{\chi}}$ 😃

→ CFT data: $a, \lambda, \underline{\Delta}$ or $b, \underline{\hat{\Delta}}$ ☹ → 😲

WHAT DO WE WANT TO DO? compute $\mathcal{F}(z,\bar{z},\omega)$ → CFT data 😃

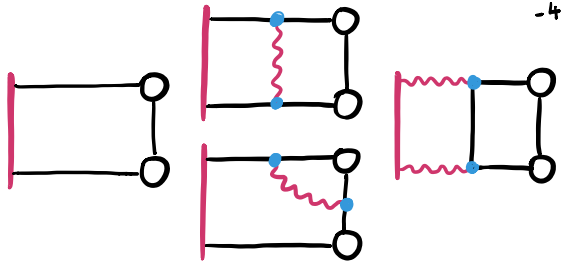
SPECTRUM OF $\chi, \hat{\chi}$

$$\mathcal{F}(z,\bar{z},\omega) = A\mathcal{G}_{\mathbb{1}} + B\mathcal{G}_{\mathcal{O}_2} + C\mathcal{G}_{\mathcal{O}_4} + \sum_{\ell} D_\ell\, \mathcal{G}_{\text{semi}} + \sum_{\Delta,\ell} E_{\Delta,\ell}\, \mathcal{G}_{\text{longs}}$$

$$\mathcal{F}(z,\bar{z},\omega) = \hat{A}\hat{\mathcal{G}}_{\hat{\mathbb{1}}} + \hat{B}\hat{\mathcal{G}}_{\hat{\mathcal{O}}_1} + \hat{C}\hat{\mathcal{G}}_{\hat{\mathcal{O}}_2} + \sum_{S} D_S\, \hat{\mathcal{G}}_{\frac{1}{4}\text{-BPS}} + \sum_{\hat{\Delta},s} E_{\hat{\Delta},s}\, \hat{\mathcal{G}}_{\text{longs}}$$

3. CFT DATA

WEAK COUPLING:
- up to NLO
- bootstrap + numerics → CFT data!



STRONG COUPLING:
- LO: $\langle \mathcal{W}_\ell \mathcal{O}_2 \mathcal{O}_2 \rangle \sim \langle \mathcal{W}_\ell \mathcal{O}_2 \rangle \langle \mathcal{W}_\ell \mathcal{O}_2 \rangle$
- next: NLO using bootstrap

EXACT RESULTS:
- remarkable limit $z = \bar{z} = \omega$

→

$$\mathcal{F}(z,\bar{z},\omega) = A\mathcal{G}^{1}_{\mathbb{1}} + B\mathcal{G}^{\pm 1}_{\mathcal{O}_2} + C\mathcal{G}^{1}_{\mathcal{O}_4} + \sum_\ell D_\ell \mathcal{G}^{0}_{\text{semi}} + \sum_{\Delta,\ell} E_{\Delta,\ell}\mathcal{G}^{0}_{\text{long}_S}$$

$$\mathcal{F}(z,\bar{z},\omega) = \hat{A}\mathcal{G}^{1}_{\mathbb{1}} + \hat{B}\mathcal{G}^{\pm 1}_{\mathcal{O}_1} + \hat{C}\mathcal{G}^{1}_{\mathcal{O}_2} + \sum_S D_S \hat{\mathcal{G}}^{0}_{\frac{1}{4}\text{-BPS}} + \sum_{\hat{\Delta},S} E_{\hat{\Delta},S}\hat{\mathcal{G}}^{0}_{\text{long}_S}$$

- $A \pm B + C = \hat{A} \pm \hat{B} + \hat{C}$ → $\hat{B}, \hat{C}$ at all-loop!
- next: $\langle \mathcal{W}_\ell \mathcal{O}_2 \mathcal{O}_4 \rangle$, $\langle \mathcal{W}_\ell \mathcal{O}_4 \mathcal{O}_4 \rangle$, ...

SUMMARY

→ kinematical form of $\langle \omega_\ell O_2(x_1) O_2(x_2) \rangle$ fixed by symmetry

→ weak coupling: bootstrap + PT + numerics → CFT data

→ strong coupling: LO → next: get NLO!

→ exact results: towards recursive relations?

THANKS!