

General Relativity from Quantum Field Theory

$$Z = \int \mathcal{D}g_{\mu\nu} e^{i \int d^4x \sqrt{-g} R}$$

Overview

$\frac{S}{\hbar} \rightarrow \infty$ Classical physics from QFT

Worldline QFT: loops become trees

Two body problem in GR & GR waves

Introduction

Quantum gravity

→ Effective field theory approach

Classical limit produces GR

Classical GR from QFT

$$S = \int d^4x \sqrt{-g} \left\{ \underbrace{\frac{R}{16\pi G_N}}_{\text{gravity}} + \sum_i \underbrace{\frac{1}{2} (g^{\mu\nu} \partial_\mu \varphi_i \partial_\nu \varphi_i - m_i^2 \varphi_i^2)}_{\text{matter}} \right\}$$

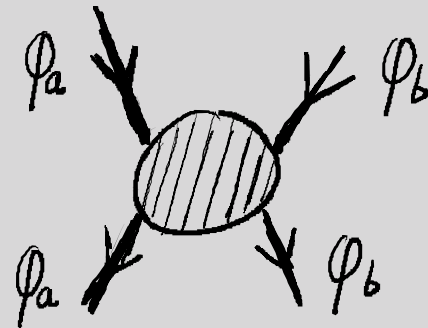
Classical limit $\hbar \rightarrow 0$: tree level? No!

$$\mathcal{L} = \sqrt{-g} \left\{ \frac{R}{16\pi G_N} + \sum_i \frac{1}{2} (g^{\mu\nu} \partial_\mu \varphi_i \partial_\nu \varphi_i - m_i^2 \varphi_i^2) \right\}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Gravitons: $\mu\nu \underset{h_{\mu\nu}}{\overset{g}{\rightsquigarrow}} \alpha\beta = \frac{1}{2} \frac{\eta^{\mu\alpha} \eta^{\nu\beta} + \eta^{\mu\beta} \eta^{\nu\alpha} - \eta^{\mu\nu} \eta^{\alpha\beta}}{g^2}$

Object of interest:



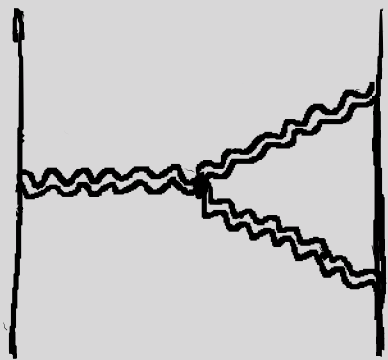
How do we let $\hbar \rightarrow 0$?

Graviton momenta scales with \hbar !

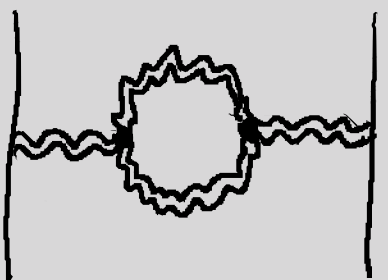
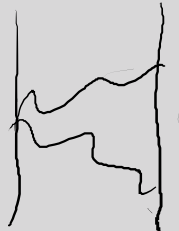
Classical: particles vs wave

$$p^\mu = \hbar k^\mu$$

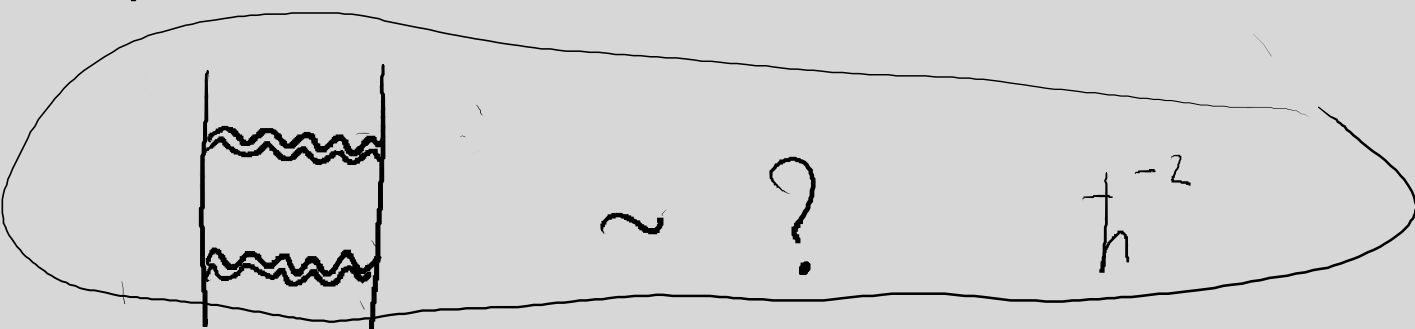
↑ ↑
momentum wavenumber



$$\sim \int d^4q \frac{1}{(q^2)^3} \frac{1}{pq} V_3 \sim q^{-1} \sim \hbar^{-1}$$



$$\sim \int d^4q \frac{1}{(q^2)^4} V_3^2 \sim q^0 \sim \hbar^0$$



$$\sim ? \quad \hbar^{-2}$$

Schwarzschild metris

$$\tilde{g}_{\mu\nu}(k) \sim \text{tree} + \text{1-loop} + \text{2-loop} + \text{3-loop} + \mathcal{O}(G_N^4)$$

The image shows a series of four Feynman diagrams representing the perturbative expansion of the graviton propagator $\tilde{g}_{\mu\nu}(k)$. Each diagram consists of an incoming wavy line with momentum k and index $\mu\nu$, and an outgoing straight line with momentum p . The diagrams are separated by plus signs. The first diagram is the tree-level propagator. The second diagram is a one-loop correction with a triangle loop of gravitons. The third diagram is a two-loop correction with a triangle loop containing two gravitons and one ghost. The fourth diagram is a three-loop correction with a triangle loop containing three gravitons. Below the diagrams is the term $+ \mathcal{O}(G_N^4)$.

Worldline QFT

- * Scalar φ becomes worldline $Z(\tau)$
- * Loops become trees

$$\int d^4x \sqrt{-g} \frac{1}{2} (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m^2 \varphi^2) \rightarrow \int d\tau m$$

With $Z(\tau)$: quantum action = classical action

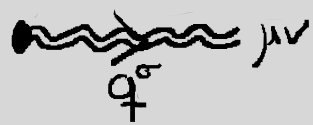
Hence: tree level = EOM
= $\hbar \rightarrow 0$

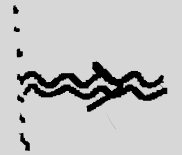
However these trees are unusual ...

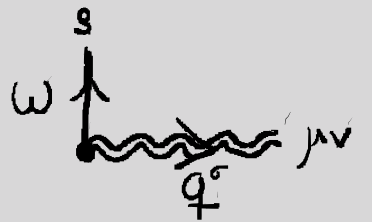
WQFT Feynman Rules

Worldlines are described by b^μ & v^μ

$$(x^\mu(\tau) = b^\mu + \tau v^\mu + Z^\mu(\tau))$$

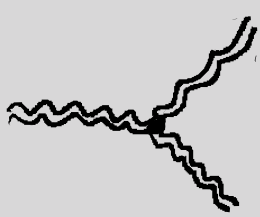

 $\sim e^{iq \cdot b} \delta(q \cdot v) v^\mu v^\nu$



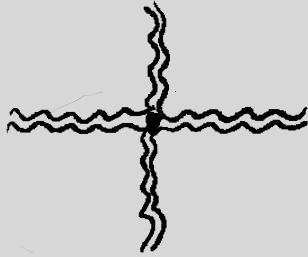

 $\sim e^{iq \cdot b} \delta(q \cdot v + \omega) v^\mu v^\nu q^s$



Bulk graviton rules as usual:



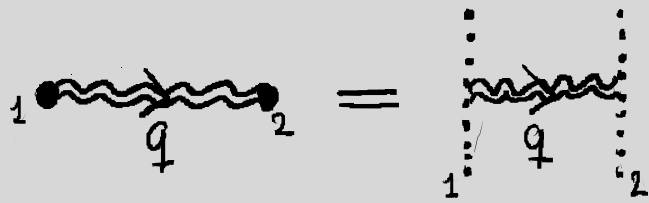
,



etc.

Example: Leading order eikonal (1PM)

(two sources with propagator)



$$\sim \int d^4 q \, e^{i q b_1} \delta(q \cdot v_1) \frac{N}{q^2} e^{-i q b_2} \delta(q \cdot v_2)$$

$$\sim N \int d^2 q_{\perp} e^{i q_{\perp} b_{\perp}} \frac{1}{q_{\perp}^2}$$

$$\sim N \ln |b_{\perp}^2|$$

Schwarzschild metric in WQFT

$$\tilde{g}_{\mu\nu}(k) \sim \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \mathcal{O}(G_N^4)$$

The diagrammatic expansion of the graviton propagator $\tilde{g}_{\mu\nu}(k)$ is shown as a sum of four terms, each representing a different order of quantum corrections:

- Diagram 1:** A single wavy line representing a graviton with momentum k and indices μ, ν .
- Diagram 2:** A graviton line with momentum k and indices μ, ν that splits into two graviton lines, which then recombine into a single graviton line.
- Diagram 3:** A graviton line with momentum k and indices μ, ν that splits into three graviton lines, which then recombine into a single graviton line.
- Diagram 4:** A graviton line with momentum k and indices μ, ν that splits into four graviton lines, which then recombine into a single graviton line.

The expansion is truncated at the fourth order, with the remainder being $\mathcal{O}(G_N^4)$.

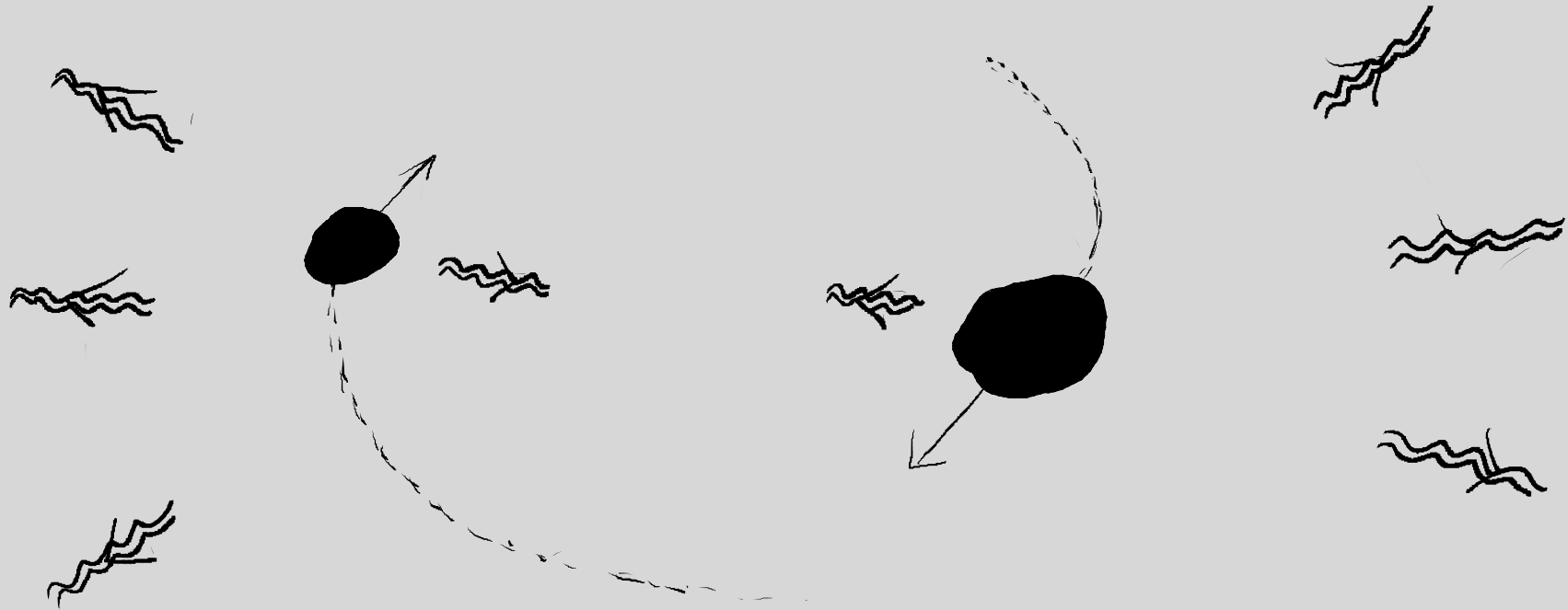
Leading order radiation: (waveform)

$$\tilde{T}^{\mu\nu} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]}$$

The diagram shows the decomposition of the leading-order radiation stress-energy tensor $\tilde{T}^{\mu\nu}$ into three terms. Each term consists of a vertical wavy line (graviton) between two horizontal dashed lines (source), connected by a horizontal solid line (particle). The radiation is shown as a wavy line extending from the vertex, labeled with $\mu\nu$.

Working on paper where we compute $\tilde{T}^{\mu\nu}$

Two body problem in GR



PN

Post Newtonian
expansion

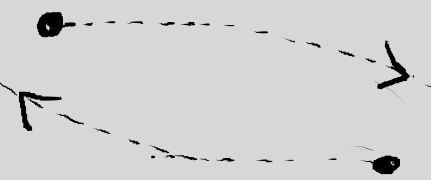


binary motion
small velocity
non-relativistic

→ Necessary for GR waveforms!

PM

Post Minkowskian
expansion



scattering
arbitrary velocity
Lorentz covariant

Conclusion

Classical GR can be derived from QFT

Loops from scalars become trees in WQFT

The Post-Minkowskian expansion is computed from QFT and describes the two body problem