General Relativity from Quantum Field Theory

$$Z = \int \mathcal{D}g_{\mu\nu}\, e^{i\int d^4x \sqrt{-g}\, R}$$

Overview

$\frac{S}{\hbar} \to \infty$ Classical physics from QFT

Worldline QFT: loops become trees

Two body problem in GR & GR waves

Introduction

Quantum gravity

→ Effective field theory approach

Classical limit produces GR

Classical GR from QFT

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G_N} + \sum_i \frac{1}{2} \left(g^{\mu\nu} \partial_\mu \varphi_i \partial_\nu \varphi_i - m_i^2 \varphi_i^2 \right) \right\}$$

gravity

matter

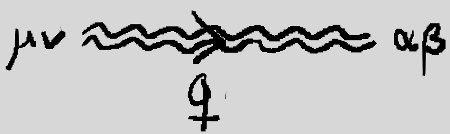
Classical limit $\hbar \to 0$: tree level? No!

$$\mathcal{L} = \sqrt{-g}\left\{\frac{R}{16\pi G_N} + \sum_i \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi_i\partial_\nu\varphi_i - m_i^2\varphi_i^2\right)\right\}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

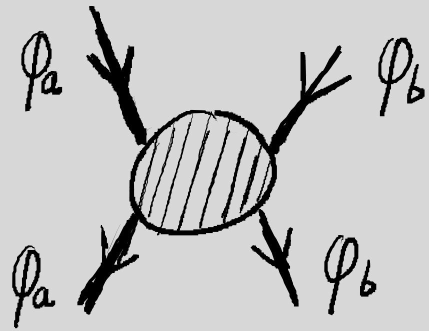
Gravitons:

$h_{\mu\nu}$



$$= \frac{1}{2}\,\frac{\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}}{q^2}$$

Object of interest:



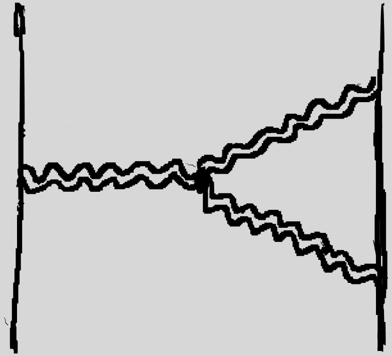
How do we let $\hbar \to 0$?

Graviton momenta scales with $\hbar$!

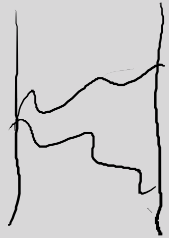
Classical: particles vs wave

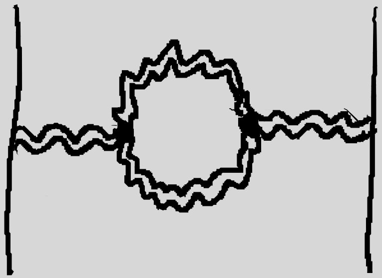
$$p^\mu = \hbar k^\mu$$

momentum   wavenumber

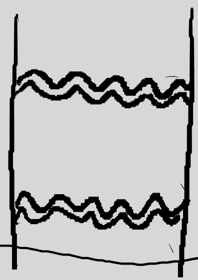


$$\sim \int \bar{d}^4 q \, \frac{1}{(q^2)^3} \, \frac{1}{pq} \, V_3 \sim q^{-1} \sim \hbar^{-1}$$





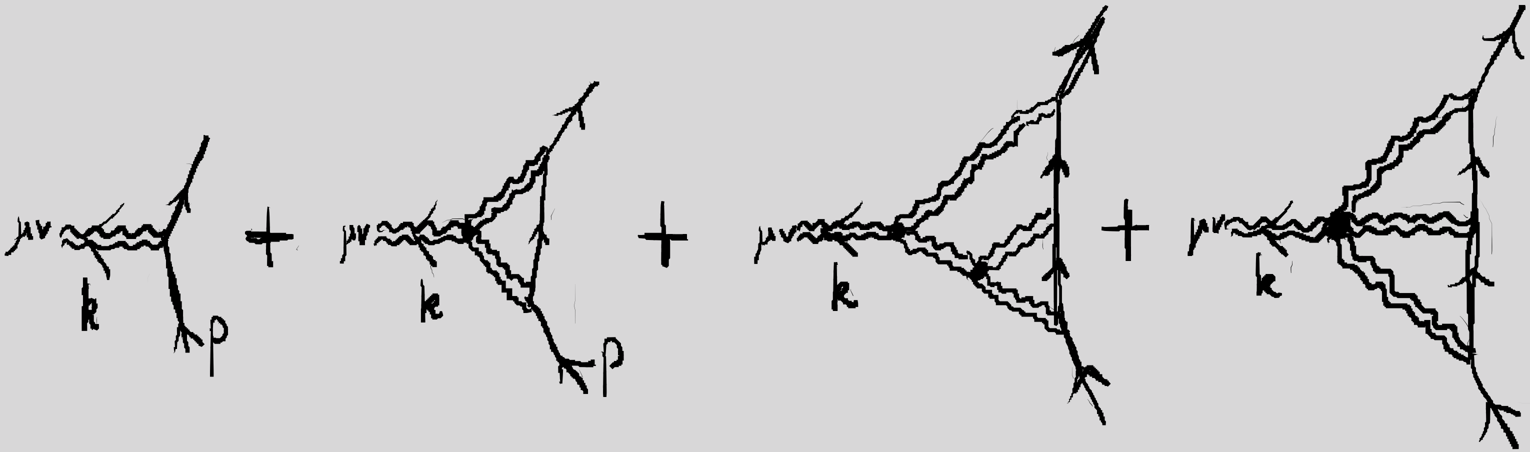
$$\sim \int \bar{d}^4 q \, \frac{1}{(q^2)^4} \, V_3^{\,2} \sim q^0 \sim \hbar^0$$



$$\sim \; ? \qquad \hbar^{-2}$$

Schwarzschild metric

$\tilde{g}_{\mu\nu}(k) \sim$



$+ \mathcal{O}(G_N^4)$

Worldline QFT

* Scalar φ becomes worldline $z(\tau)$
* Loops become trees

$$\int d^4x \sqrt{-g}\, \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - m^2\varphi^2\right) \rightarrow \int d\tau\, m$$

With $Z(\tau)$: quantum action = classical action

Hence: tree level = EOM

$= \hbar \to 0$

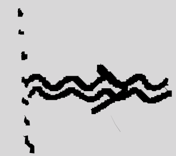
However these trees are unusual ...

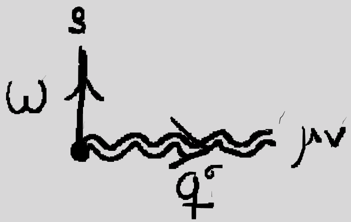
WQFT Feynman Rules

Worldlines are described by b^μ & v^μ

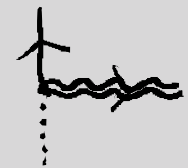
$$\left(x^\mu(\tau) = b^\mu + \tau v^\mu + Z^\mu(\tau) \right)$$

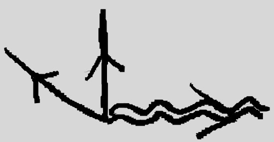
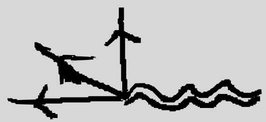
$$\sim \quad e^{iq\cdot b}\,\delta(q\cdot v)\, v^\mu v^\nu$$



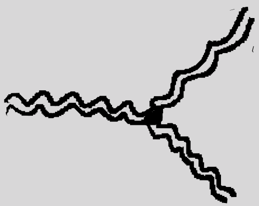
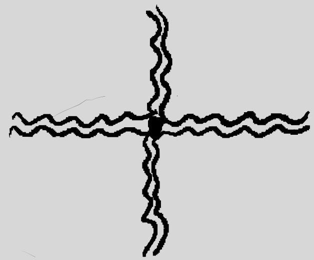


$$\sim \quad e^{iq\cdot b}\,\delta(q\cdot v+\omega)\, v^\mu v^\nu\, q^s$$



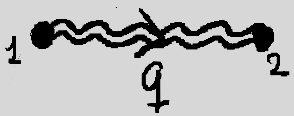
 ,  ...

Bulk graviton rules as usual:

 ;  etc.

Example: Leading order eikonal (1PM)

(two sources with propagator)



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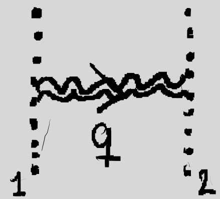


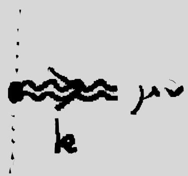
$$\sim \int \bar{d}^4 q \; e^{iqb_1} \delta(qu_1) \frac{N}{q^2} e^{-iqb_2} \delta(qu_2)$$

$$\sim N \int \bar{d}^2 q_\perp \; e^{iq_\perp b_\perp} \frac{1}{q_\perp^2}$$

$$\sim N \ln |b_\perp^2|$$

Schwarzschild metric in WQFT

$\tilde{g}_{\mu\nu}(k) \sim$



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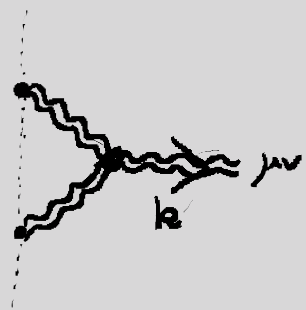

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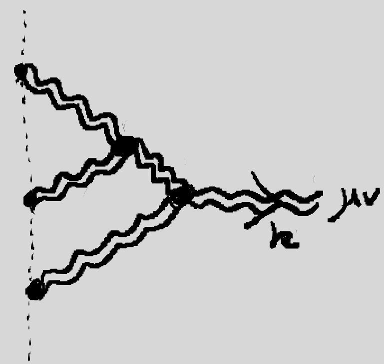

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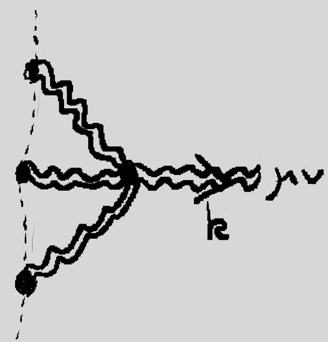
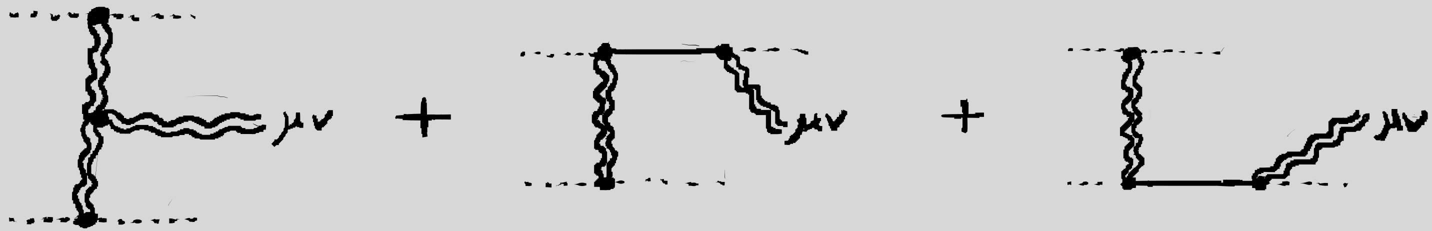


$+ \mathcal{O}(G_N^4)$

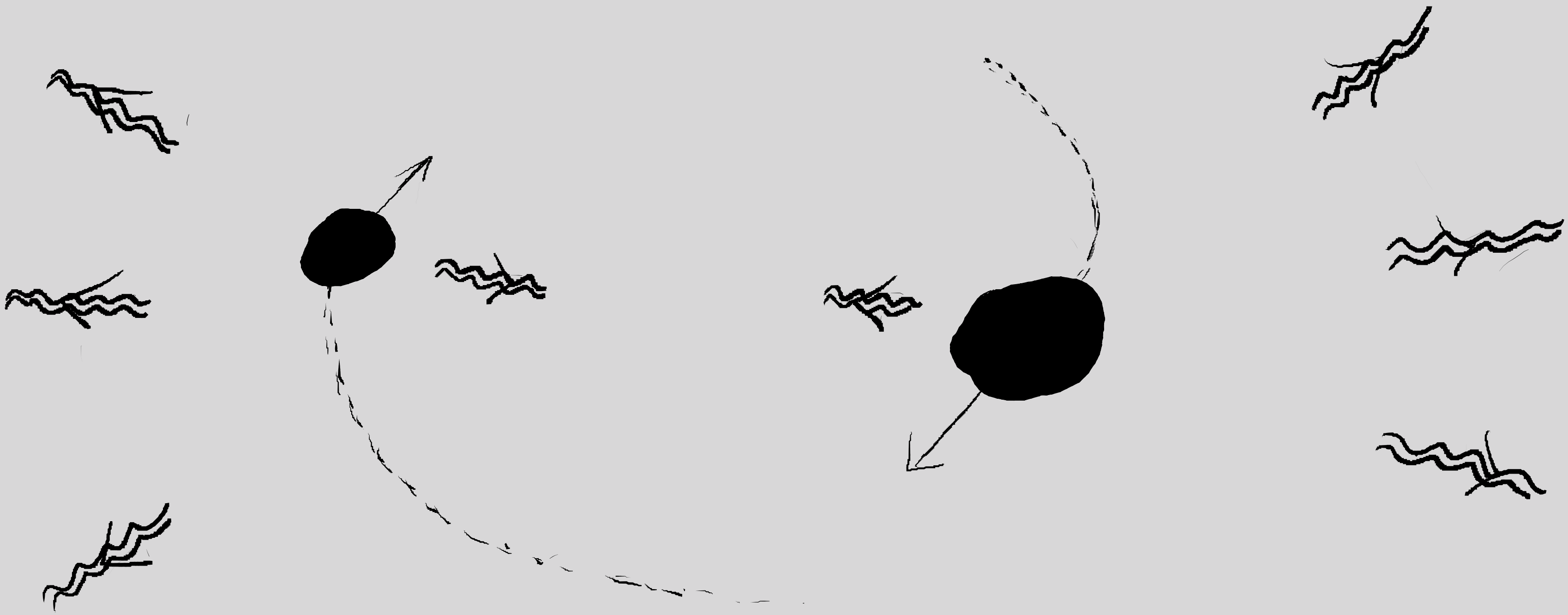
Leading order radiation:
(waveform)

$\tilde{\tau}^{\mu\nu} =$



Working on paper where we compute $\tilde{\tau}^{\mu\nu}$

Two body problem in GR

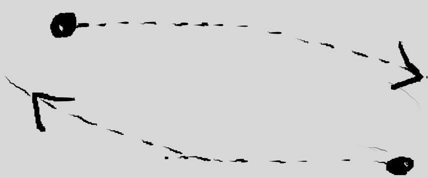


PN Post Newtonian expansion

binary motion
small velocity
non-relativistic

→ Necessary for GR waveforms!

PM Post Minkowskian expansion



scattering
arbitrary velocity
Lorentz covariant

Conclusion

Classical GR can be derived from QFT

Loops from scalars become trees in WQFT

The Post-Minkowskian expansion is computed from QFT and describes the two body problem