



RTG 2575:

Rethinking  
Quantum Field Theory

# Loop Tree Duality

explained to my grandma\*  
\* supposing she were a PhD student in Physics

05.01.2021

STUDENT SEMINAR BY DANIELE ARTICO





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# Numerical Integration: WHY?

- More precise experiments need **BETTER** calculations
- Actual scattering processes: at least **NNLO** QCD, many **mass** scales



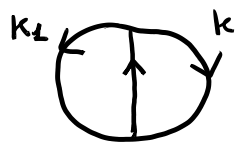
**NUMERICAL INTEGRATION**

some issues:  $D$ -dimensions, singular integrands ...





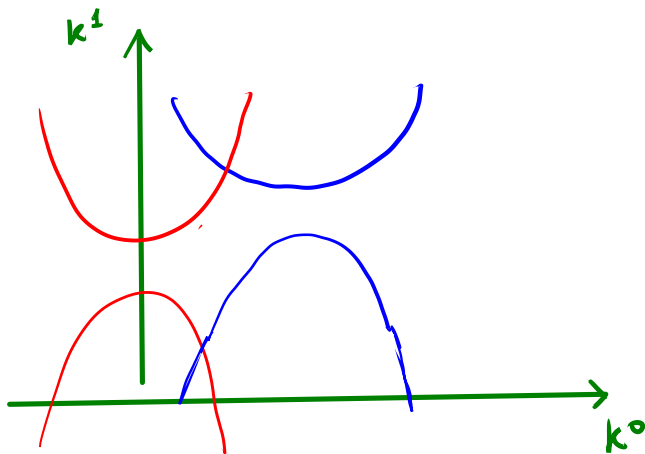
# Let's start!



$$\sim \int d^D k_1 d^D k_2$$

$$\frac{1}{(k_1^2 - m^2 + i\epsilon)(k_2^2 - m^2 + i\epsilon) [(k_1 + k_2)^2 - m^2 + i\epsilon]}$$

for numerical:  $D$  integer,  $\epsilon = 0$



to avoid singularities we use contour deformation

- non compact surfaces
- intersections
- one loop ✓ multi loop ✗

simulates  $+i\epsilon$



$$k' = k + i\tilde{k} \rightarrow (k'^2 - m^2) = (k^2 - m^2 + 2i\tilde{k} \cdot k)$$



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# Loop Tree Duality

We can use the **RESIDUE THEOREM** <sup>with  $+i\epsilon$</sup>  on the Energy components

$$\int dk_0 \frac{1}{(k_0^2 - E^2)} = -2\pi i \delta^+(k_0^2 - E^2)$$

This means we are considering integration **ON MASS SHELLS**

We get **PHASE SPACE** integrals  $\rightarrow$  IR divergencies can cancel at integrand level!

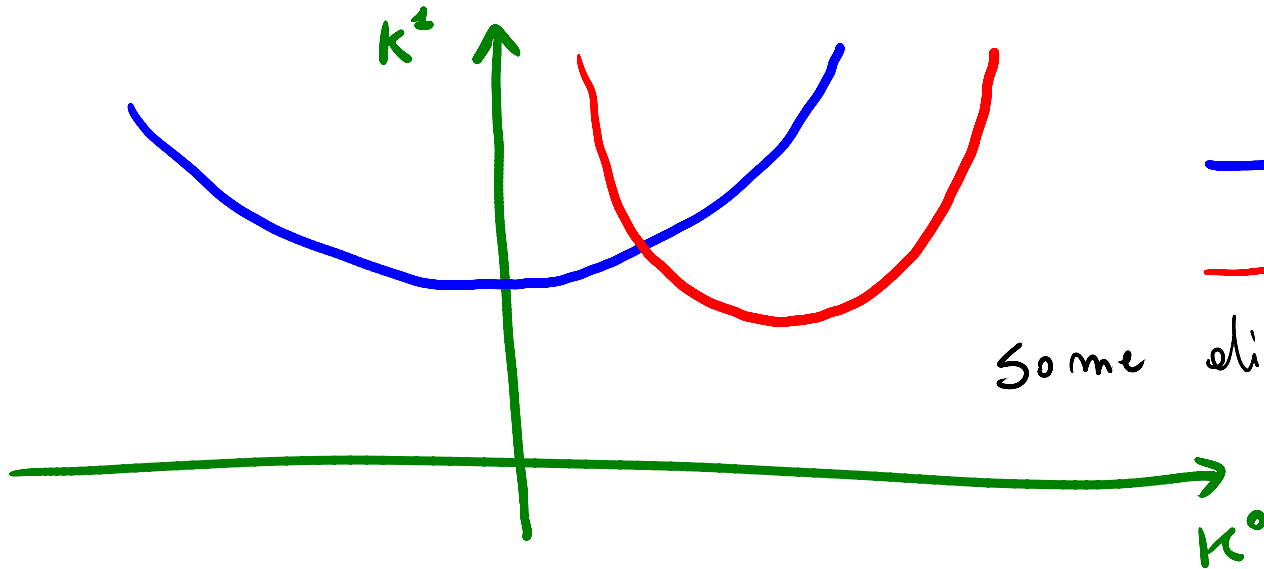




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# The importance of living on a shell



— OUT to IN

— IN to OUT

Some divergencies will cancel ...





# Mested Residues

Let's go back to

$$\begin{array}{c}
 \text{Diagram: a circle with a vertical line through the center, two external legs on the left labeled } k_1 \text{ and } k_2 \\
 \sim \int d^D k_1 d^D k_2 \frac{1}{\underbrace{(k_1^2 - m^2 + i\epsilon)}_{D_1} \underbrace{(k_2^2 - m^2 + i\epsilon)}_{D_2} \underbrace{[(k_1 + k_2)^2 - m^2 + i\epsilon]}_{D_3}}
 \end{array}$$

We have to cut **2 ordered legs**

- |          |        |   |
|----------|--------|---|
| • 1, 2 ✓ | • 1, 3 | } they combine eliminating uncertain residues coming after $\int d^D k_i$ |
| • 3, 2 ✓ | • 3, 1 |   |

→ one term each **SPANNING TREE** (1906.06138 hep-ph)





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# To do List

- understand the singularities } next time
- build contour deformation } next time
- treat UV and IR divergences } long term goals
- some phenomenology:  $e^+e^- \rightarrow t\bar{t}H$  } long term goals

