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Deriving the Gravitational Constant Within Modern Cosmology

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Abstract The gravitational constant G is a free parameter in both Newtonian mechanics and general relativity; it is not derived from fundamental magnitudes within those theories but rather is a fundamental natural constant. Robert Henry Dicke proposed a reduction of the gravitational constant G to global cosmological magnitudes. However, he was unable to satisfy his claim in terms of a provable equation. We prove how Dicke's claim can be satisfied based on modern cosmology by deriving an equation that reduces G to its causally relevant global cosmological magnitudes. Using this equation, we identify some philosophically significant consequences and applications.

1 Introduction

In this paper, we address the question of how to derive the gravitational constant G on the basis of some standard assumptions of modern cosmology. We assume that the visible universe is homogeneous, isotropic, spherical, (roughly) flat and expanding at an accelerating rate due to dark energy. In the following, we refer to these assumptions as "the standard assumptions". We neither discuss these assumptions nor do we intend to discuss *different* possible ways to reduce G according to *different* assumptions concerning the geometry of space-time or *different* cosmological models. Instead, we wish to demonstrate, in the most simplified way, the general possibility of reducing G to those magnitudes that determine the global cosmological conditions of gravitation in our universe according to the presumed standard assumptions. We do this in proving an equation that equates G with global cosmological conditions according to modern cosmology. The proof is rather trivial and the idea of reducing G to global cosmological conditions is well known. Thus, although the equation that we demonstrate (equation (8)) is not mentioned in the literature to our

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knowledge, we do not claim to show something essentially new. However, we think that the fact that G is *derivable* from standard assumptions of modern cosmology is rarely recognized, nevertheless significant. It is important to see that Dicke's idea of reducing G is realizable within modern cosmology and to consider its cosmological implications and philosophical consequences. Unlike in Newtonian mechanics, general relativity (GR) or other cosmological models, G should no longer considered as a free parameter within modern cosmology. This seems to us philosophically of the greatest interest, whereas the physical question of how to do justice to such an account of G on the basis of more general assumptions remains open. In particular, we do not contribute to the question of conceptualizing G as a variable depending on exact mass distribution within the universe and on the specific curvature of space-time.

To understand our aim, it is important to distinguish the questions of (i) how to *measure* G within local physics and (ii) how to *reduce* G to global magnitudes. According to our understanding, reducing G to global magnitudes means defining G in terms of an equation referring to the distribution of mass in the universe. By contrast, measuring G requires an empirical means of determining the value of G. Which quantities can be measured or calculated depend on empirical conditions, whereas which magnitudes can be reduced and which are fundamental depend on the relevant theory. In general, the question of how to reduce physical magnitudes is different from the question of how to determine their values through empirical measurement. Measuring a pressure using a barometer, for example, is not equivalent to defining pressure in terms of any property of the barometer. Likewise, defining pressure as the force per area does not necessarily prescribe how it can be measured. Thus, we do not claim that the well-known problem of how to determine the value of G can be solved by defining G in terms of global cosmological conditions. Instead, we strive to answer the intricate question of how to explain what gives rise to the specific value of G.

2 Dicke's Coincidence

According to GR, the acceleration of material particles is induced by nothing less than the interaction of all particles in the universe. Contrary to Newton's mechanics, there is no acceleration with respect to absolute space. Space, its geometry and its properties are not independent of matter. Gravitation is a property of space-time. It is not something that arises from matter in a given independent space.

Considerations regarding attempts to relate physical space to matter can be traced back to the writings of Bishop Berkeley. The most prominent advocate of a matter-based conception of space was the Austrian physicist and philosopher Ernst Mach, to whom the so-called *Mach's principle* is attributed. Its exact definition and its consequences are controversial and will not be discussed here.[1] In the following, we concern ourselves only with Robert Henry Dicke's relevant interpretation of Mach's principle; cf., e.g., Dicke (1959b), p. 34-37; Dicke (1959a), p. 622; and Brans and Dicke (1961). Brans (2005), p. 8[2], dubbed this interpretation "Dicke's form of Mach's principle" and briefly described it as follows:

> The gravitational constant [...] should be a function of the mass distribution in the universe.

Neither Brans nor Dicke drew a distinction between "mass" and "matter". Thus, "mass" refers to "matter" in this quotation. Hence, G, or, more precisely, the value of G for any point P in space-time, should be a function of the matter of all material particles within the visible universe that is causally related to P. Dicke regarded his form of Mach's principle as a principle that determines "physically meaningful solutions of Einstein's field equations" (Dicke (1959b), p. 36).[3] For example, Mach's principle rules out a de Sitter universe that does not contain matter or a Gödel universe as it is not isotropic due to the rotation. However, other models satisfy both the field equations and this form of Mach's principle. We will demonstrate this for Einstein's universe below (cf. p. 4) and for the standard assumptions in section 3.

According to Dicke's form of Mach's principle, gravitation is not merely a local property but rather is "determined by distant matter" (Dicke (1959b), p. 35). Brans and Dicke were dissatisfied with the fact that "although in general relativity spatial geometries are affected by mass distributions, the geometry is not uniquely specified by the distribution" (Brans and Dicke (1961), p. 925). They sought an expression that would show how gravitation and the geometry of space are specifically determined by the distribution of matter in the universe. Increasingly accurate measurements of cosmological data led Dicke to the following coincidence in the late 1950s:[4]

$$G \approx c^2 \, \tfrac{R}{M} \tag{1}$$

Here, G is the gravitational constant, c is the speed of light in vacuum, M is the matter contained in the "visible (i.e., causally related) universe" (Brans and Dicke (1961), p. 926), and R is the radius of the visible (observable) universe. We call (1) "Dicke's coincidence". Dicke conceived this coincidence as a realization of Mach's principle, insofar as G depends on $\frac{R}{M}$; cf. Dicke (1959a), p. 622, and Brans and Dicke (1961), p. 925. However, (1) is merely a

[1] Bondi and Samuel (1997) distinguish 11 interpretations of Mach's principle; cf. also Barbour and Pfister (1995) as well as Dambmann (1990). In many respects, our account is similar in spirit to that of Essen (2013).

[2] Peebles (2016) provides a nice overview on Dicke's study of gravity and discusses his understanding of Mach's principle in section 4.1, cf. also Lynden-Bell (2010), section 3.

[3] Einstein (1923) already argued similarly, cf. also Peebles (2016), p. 12: "Following Einstein (1923), some take Mach's Principle to be that a philosophically acceptable universe is described by a solution of Einstein's field equation in which inertial motion everywhere is determined solely by the distribution and motion of matter everywhere."

[4] Cf., e.g., Dicke (1959b), p. 36; Dicke (1959a), p. 622; and Brans and Dicke (1961), p. 926. Cf. also Sciama (1953) for an even earlier relevant discussion.

coincidence. Dicke notes that (1) neither follows from GR nor is incompatible with GR (cf. Dicke (1959b), p. 36). In particular, Dicke's coincidence is not derivable from any cosmological theory, it is only very roughly consistent with the present data, and it is incompatible with the standard assumptions.

A simple example of a cosmological model that allows one to derive a formula for G that satisfies Dicke's form of Mach's principle is Einstein's static, homogeneous and spherical universe. In this model, the following equation for the radius R as a function of the total amount of mass within the visible universe is derivable (cf. Kragh (2011), p. 39):

$$R = df M \frac{\kappa c^2}{4\pi^2} \tag{2}$$

"$df M$" refers to the total amount of mass corresponding to all forms of energy, not only to the mass of matter, M. Einstein did not, as is usual in the present day, refer to "dark energy" as separate from the energy of matter. However, he was obliged to introduce the cosmological constant Λ to harmonize his static model of the universe and his field equations. This is why we use "df" (the "dark factor") as the factor by which M must be multiplied to equal the total mass of the visible universe.

Einstein's gravitational constant κ is defined as $\frac{8\pi G}{c^4}$. Thus, the following equation for G can be derived (cf. Barrow (2012), p. 320, footnote 13):

$$G = \frac{\pi}{2\, df} c^2 \frac{R}{M} \tag{3}$$

This equation satisfies Dicke's form of Mach's principle because it states that G depends on $\frac{R}{M}$. Furthermore, (3) is derivable. However, Einstein's universe is incompatible with modern cosmological models that imply expansion and/or flatness of the universe. In the following, we show how to derive an equation for G that (i) states that G depends on $\frac{R}{M}$, thereby satisfying Dicke's form of Mach's principle; (ii) follows from Einstein's field equations; and (iii) presumes the standard assumptions.

3 Proof

3.1 Basic Assumptions

Our proof is based on Friedmann's first equation:[5]

[5] Kragh (2015), p. 6 mentions the possibility of deriving "a relation of this kind [i.e. Dicke's coincidence] [...] from ordinary Friedmann cosmology". However, he does neither show in detail how to do this nor does he consider differences to Dicke's coincidence or implications of such a derivation such as the assumption of dark energy. Peebles (2016), p. 11 also alludes to deriving an equation similar to (1) from "the relativistic Friedmann-Lemaître cosmology". However, he neither considers the implication of dark matter (in contrast to "observable matter", Peebles (2016), p. 11) and dark energy nor does he derive an equation that refers to the distribution of mass within the visible universe. Instead, his equation only refers to the radius of the Hubble sphere. From our point of view, an equation has to be proven that

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\,\pi\,G}{3}\rho_M - \frac{k\,c^2}{a^2} + \frac{\Lambda\,c^2}{3} \quad (4)$$

where $\dot{a} = \frac{da}{dt}$.This equation governs the expansion of space in isotropic and homogeneous models. It is fundamentally derived from Einstein's field equations and is part of nearly all modern cosmological models. H is the Hubble parameter, Λ is the so-called cosmological constant, k is the factor of curvature, ρ_M is the matter density, and a is the scale factor, whose value is defined as 1 for the present age t_u of the universe. The energy density ρ is equal to the sum of the matter density, ρ_M, and the dark energy density, ρ_Λ. We abstain here from considering the comparatively small energy of radiation. As is usual, we also abstain from characterizing H, ρ_M, a and Λ in (4) as functions of time.

Given the flatness of the universe, the factor of curvature k is equal to 0. This widely accepted assumption is empirically validated by the cosmic microwave background (CMB), which is not curved either inward or outward. This is highly significant because it makes it possible to derive the following equation for the critical density $\rho_c(t)$ that is required for a flat universe:

$$\rho_c(t) = \frac{3\,H(t)^2}{8\,\pi\,G} \quad (5)$$

(5) is derived from (4) using $\rho_M = \rho - \rho_\Lambda$ and $\rho_\Lambda = \frac{\Lambda\,c^2}{8\,\pi\,G}$ in addition to $k = 0$. ρ_c in (5) refers to the total energy, not only the energy of matter. (5) is a fundamental equation of modern cosmology.

The critical density of the present universe corresponds to approximately 5 hydrogen atoms per cubic meter. However, the average density of visible matter corresponds to only approximately 0.2 hydrogen atoms per cubic meter on large scales. Even the sum of the visible and dark matter is still too small, by a factor of $\frac{1}{0.32} \approx 3.1$, to account for the critical density of the present universe. This is one reason why it is assumed that approximately 68% of all energy in the universe is currently dark energy. These fractions of the energy due to visible matter ($= \Omega_b$) and dark matter ($= \Omega_m$), their sum Ω_M, and the fraction due to dark energy ($= \Omega_\Lambda$), all varying with the age of the universe, are parameters of modern cosmologies such as the Λ-CDM model, which is the standard model of modern cosmology. According to modern cosmologies, the ratio of the amount of dark energy to the total energy, i.e., Ω_Λ, is a fundamental magnitude for explaining cosmic events.

Originally, the expansion of the universe was expected to be slowing down because of gravity. To general surprise, however, it was calculated in 1998, based on observations of supernovae in distant galaxies, that the universe is expanding with an increasing velocity at its edges. This acceleration is also explained by dark energy, which counteracts gravity.

The Hubble parameter $H(t)$ specifies the dependence of this expansion on time and distance. It is difficult to determine this parameter exactly. Its present

identifies the causal relevant factors of G within the framework of modern cosmology. It neither suffices to stipulate a coincidence nor does it suffice to prove some equivalence for G.

value, the so-called "Hubble constant" H_0, is approximately 67.11 ±0.77 km/s per mpc (1 mpc = 1 megaparsec = $3.26 \cdot 10^6$ light-years), according to the data from the ESA's Planck mission that were released in 2015. This means that galaxies at a distance of one megaparsec are moving away with a velocity of 67.11 ±0.77 km/s. The farther away a galaxy is, the faster it is moving away. A sphere of radius $\frac{c}{H(t)}$ $(= R_H(t))$ is called a "Hubble sphere". The velocity of expansion at the edge of a Hubble sphere is exactly c, i.e., the maximum speed at which gravity can be transmitted according to relativity. We regard $R_H(t)$ as a "norm" of the universe's expansion. If R_H were constant, then the radius R of the visible (observable) universe would be identical to R_H and the Hubble sphere would be identical to the "causally related universe"; see the quote above on p. 3 from (Brans and Dicke (1961), p. 926. However, $R(t_u)$ for the visible universe is ≈ 46.6 light-years at its present age t_u, whereas $R_H(t_u)$ is ≈ 14.2 light-years. Thus, $R(t_u) > R_H(t_u)$ because $H(t)$ is not constant according to modern cosmology; instead, $H(t)$ converges to a constant. Hence, the visible universe is expanding at an accelerating rate, currently with a superluminal velocity at its edge. It should be noted that neither R_H nor R is identical with to radius of the so-called event horizon. The event horizon describes the sphere of future events that are visible; by contrast, the visible universe describes the sphere of past events that are visible. Both spheres, in turn, are normalized with respect to the radius of the Hubble sphere. Our considerations concern the gravitational influence of mass that is transmitted at the speed of light from the past and, thus, the visible universe (and not the event horizon).

The cosmological constant Λ considers the time-dependent acceleration of the expansion and, thus, the ratio of the dark energy to the total energy within the visible universe. Its current value is approximately $1.07 \cdot 10^{-52} m^{-2}$ according to the Λ-CDM model. Its value decreases with the continuing expansion of the universe.

3.2 Derivation

In the following, we assume a flat, homogeneous and spherical universe. These assumptions are standard in modern cosmology. We begin our derivation from the critical energy density as given by equation (5): $\rho_c(t) = \frac{3\,H(t)^2}{8\,\pi\,G}$. Because of its flatness, space is Euclidean. Thus, the volume V_s of a sphere is given by the following equation:

$$V_s = \tfrac{4}{3}\,\pi\,R^3 \tag{6}$$

Consequently, the mass density ρ of a homogeneous sphere is equal to $\frac{df(t)M(t)}{\frac{4}{3}\pi R(t)^3}$. We represent the sum of the mass of (visible + dark) matter and dark energy by $df(t)\,M(t)$. Within the ΛCDM model, the dimensionless "dark factor" $df(t)$

depends on time as the universe's expansion accelerates.[6] At present, this factor is equal to approximately 3.14 according to the Λ-CDM model and is slowly increasing. The amount of matter also depends on time because matter may appear or disappear at the horizon of the visible universe.

This yields the following equation, now written with explicitly time-dependent parameters:

$$\frac{df(t)M(t)}{\frac{4}{3}\,\pi\,R(t)^3} = \frac{3\,H(t)^2}{8\,\pi\,G} \tag{7}$$

The left-hand side describes how the density of a sphere depends on its expanding radius as a function of time. The right-hand side describes the density of an expanding, flat and homogeneous sphere as a function of the Hubble parameter.

(7) still holds for an expanding sphere in general. Therefore, this equation also holds for the flat visible universe. Hence, we identify $M(t)$ and $R(t)$ with the matter and radius, respectively, of the visible universe because we wish to relate G to the sphere of causally related matter. Utilizing $H(t) = \frac{c}{R_H(t)}$ and isolating G results in the following equation:

$$G = (\frac{R(t)}{R_H(t)})^2 \frac{1}{2\,df(t)} c^2 \frac{R(t)}{M(t)} \tag{8}$$

We call (8) the "G-equation". To our knowledge, this specific equation is not considered in the literature. Similar to Dicke's coincidence as expressed in (1) and to equation (3) for Einstein's universe, in this equation, G depends on $\frac{R(t)}{M(t)}$. Hence, it satisfies Dicke's form of Mach's principle and dictates that G depends on the distribution of matter in the visible universe. As in (1) and (3), the speed of gravity c appears on the right-hand side of (8). In contrast to (1) and (3), however, the dimensionless parameters $(\frac{R(t)}{R_H(t)})^2$ and $df(t)$ also appear on the right-hand side of (8), and the values of R and, consequently, M depend on time. This is because the universe is undergoing accelerating expansion, and this acceleration is caused by dark energy. This calls for these dimensionless parameters as correction factors to adjust the specific expansion behavior of the universe.

4 Consequences

In this section, we first draw some general conclusions and then apply the derived equation (8) to reformulate several prominent equations. Our considerations in this section concern the *interpretation* of the derived equation (8) and are intended to stimulate debate on the nature of the gravitational constant.

[6] Baryshev and Teerikorpi (2012), p. 141, use the notation $\alpha(S)$ instead of $df(t)$. However, we think df is more convenient.

4.1 General Consequences

The G-equation given in (8) makes it possible to reduce G to a mass distribution that is, on large scales, homogeneous within a flat and expanding universe. Thus, G is eliminated as a natural constant. In this respect, G differs from the speed of light c and Planck's constant h, which specify the maxima and minima of causal processes according to special relativity and quantum mechanics. In contrast to c and h, there seems to be no internal reason for assuming a gravitational constant. G seems to be nothing more than a proportionality constant. For this reason, Newton avoided the use of a gravitational constant. In *Principia*, he performed his calculations in terms of ratios that allowed him to cancel out G in his equations; cf. Gribbin (1998), p. 12. However, Newton could not provide a general formula for G. By utilizing (8), however, it becomes possible to realize the intention to avoid the use of G in cosmology.

One might question the value of reducing a natural constant on the basis of the empirical and inexact assumption of flatness and on the basis of vague values such as $M(t)$ and $R(t)$. However, our intent is neither that the value of G should be determined through the use of (8) nor that (8) should serve as some sort of theory- or model-independent reduction. Instead, we wish to argue that G is not a natural constant that is given a priori but rather should be understood against the background of a cosmology.

It is important to note that reducing G to the entities on the right-hand side of (8) is not a circular argument. This is true despite the fact that the amount of matter cannot be estimated independent of a given value of G. However, as explained above on p. 2, the task of measuring, and thus determining the value of, G is independent of that of tracing back from G to identify the global magnitudes that give rise to its value.

(8) identifies the global cosmological magnitudes on which G fundamentally depends. It thus allows one to non-arbitrarily define, and thus to explain, an otherwise arbitrary natural constant. R and M are the causally relevant factors that determine the value of G. c is the speed of gravity. The sphere of causally interacting events is confined to the expanding visible universe because gravity can be transmitted no faster than the speed of light. The remaining dimensionless factors are related to the expansion of the homogeneous and flat universe for which G constantly holds. Local inhomogeneous matter distributions interact against the background of the fundamental cosmological magnitudes, and these inhomogeneities cause local curvatures of space. One might compare the relation of a flat and homogeneous universe to gravity to the relation of darkness to optics: Just as darkness serves as a homogeneous background to optical events, so too does flatness to gravitational events.

Rewriting (8) yields the following equation (from here on, we abstain from explicitly denoting time-dependent magnitudes as functions of time):

$$\frac{G}{c^2} = \left(\frac{R}{R_H}\right)^2 \frac{1}{2df} \frac{R}{M} \tag{9}$$

$\frac{G}{c^2}$ is the factor that represents the extent to which the geometry (or curvature) of space is locally affected by the presence of matter. (9) reveals that the geometrical properties of curved space depend on the ratio $\frac{R}{M}$.

Although the mass of dark energy and the radius of the visible universe are functions of time according to modern cosmology, (9) indicates that the right-hand side of (9) remains constant as long as G and c are constant. The expansion of mass (including dark energy) and the expansion of space occur at a constant ratio. In this respect, mass and space are not independent; there is no expansion of space without expansion of mass.[7] According to modern cosmology, this expansion of mass relies on the expansion of the visible universe and, thus, on the expansion of dark energy in the first place; no creation of matter is assumed. G's constancy is not given a priori. Instead, it depends on the special properties of the geometry of space-time on global scales and, thus, on the critical density. According to the standard assumptions, G's constancy is a consequence of the constant flatness of the visible universe. For other cosmological models in which different assumptions are adopted concerning the geometry of space-time, it may well be questioned whether G is constant.

Furthermore, as the gravity potential decreases only as a factor of $\frac{1}{R}$ but the number of stars within the visible universe increases roughly as R^2, distant matter dominates the total gravity potential at any arbitrary point in the visible universe; cf. Sciama (1959), p. 120f. Thus, local inhomogeneities exert no significant influence on G.[8] The fact that distant matter stabilizes space strongly, also explains why gravitational waves are difficult to detect and measure.

4.2 Application to Equations

4.2.1 GR Equations

In the following, we show how equation (9) can be utilized to rewrite equations of GR. We do not argue that one *should* eliminate $\frac{G}{c^2}$ within equations of GR. After all, this elimination implies specific assumptions of a cosmological model that goes beyond GR. However, we argue that the elimination of $\frac{G}{c^2}$ in this way enhances the explanatory power of a cosmological model that contains both (9) and the equations of GR.

[7] Dicke (1959b), p. 37, states that "the structure of the universe may be such that the ratio M/R does not change with time". However, he assumed Λ to be 0 "until its existence is forced by observations" (Dicke (1963), p. 507). Thus, he was not inclined to postulate an entity such as dark energy. Therefore, the constancy of $\frac{M}{R}$ was either inconsistent with an expanding universe or one was obliged to postulate the creation of matter. According to the standard assumptions, however, the structure of the universe can be identified by its flatness, which is determined by the critical energy density.

[8] Essen (2013), p. 144, argues similarly and concludes nicely:

> [...] inertia has an intrinsic non-local nature. It is thus difficult to investigate by local measurements – the main reason that these matters remain obscure and intimately connected to cosmology.

According to GR, the following equation holds:

$$\delta = 4\,G\,\frac{m}{r\,c^2} \tag{10}$$

This is Einstein's famous formula for the bending of light δ that is induced by a gravitational lens of mass m and radius r. Using (9), equation (10) can be rewritten as follows:

$$\delta = (\frac{R}{R_H})^2\,\frac{2}{df}\,\frac{m}{M}\,\frac{R}{r} \tag{11}$$

In this formula for δ, matter and radius appear in a direct ratio. The influence of all relevant mass in the universe on the mass m is directly expressed in this formula. The factors $\frac{m}{M}\frac{R}{r}$ that are causally relevant to the bending of light δ are represented. The fundamental idea of GR that gravity is a property of space-time that is determined by mass is directly expressed.

According to GR, the following equation also holds:

$$r_s = 2\,m\,\frac{G}{c^2} \tag{12}$$

This is the formula for the Schwarzschild radius r_s of a body of mass m. To emit light, the size of a body of mass m must exceed this radius. The Schwarzschild radius is identical to the radius of the event horizon of a static black hole. From (9), the following formula is obtained:

$$r_s = (\frac{R}{R_H})^2\,\frac{1}{df}\,\frac{m}{M}\,R \tag{13}$$

This is the new formula for the Schwarzschild radius r_s of a body of mass m. Again, the mass m appears in a ratio with respect to the total amount of matter M from which gravity originates. The relevant factors R and M are not hidden behind constants but rather appear explicitly.

Using (9), Einstein's gravitational constant κ can be defined in a new way:

$$\kappa = \frac{8\,\pi\,G}{c^4} = (\frac{R}{R_H})^2\,\frac{4\pi}{df\,c^2}\,\frac{R}{M} \tag{14}$$

One can use this definition in Einstein's field equations. Their general form is as follows:

$$G_{\mu\nu} = \kappa\,T_{\mu\nu} \tag{15}$$

$G_{\mu\nu}$ is the geometric tensor that represents the gravitational field in terms of the geometric properties of space-time. $T_{\mu\nu}$ is the energy-impulse tensor that determines the properties of space-time. From (14), one can see that κ is more than a simple relational constant. Replacing κ in accordance with (14) results in the following equation:

$$(\frac{R}{R_H})^2\,\frac{1}{4\pi R}G_{\mu\nu} = \frac{1}{df\,M\,c^2}\,T_{\mu\nu} \tag{16}$$

The factor $(\frac{R}{R_H})^2\frac{1}{4\pi R}$ on the left-hand side determines the space of the visible universe, whereas the factor $\frac{1}{df\,M\,c^2}$ on the right-hand side determines the energy of the visible universe. According to this representation, the space on the left-hand side and the energy-impulse tensor on the right are normalized through the factorization of κ. Again, a fundamental equation of GR obtains an obvious meaning through the replacement of incomprehensible constants as a consequence of applying (9).

4.2.2 Λ Formula

Based on the standard assumptions and the assumption that dark energy fills all space, one can also use (9) to obtain a manifest formula for the cosmological constant Λ.

We start from the assumption that the total energy is identical to the sum of the matter energy and the dark energy. According to our standard assumptions, space is homogenous, isotropic and flat. Thus, energy is evenly distributed. On this basis, we may apply Einstein's formula $E = m\,c^2$ without referring to a tensor and identify the total energy E with $df M\,c^2$. Thus, the following holds for the dark energy E_Λ:

$$E_\Lambda = (df - 1)\,M\,c^2 \tag{17}$$

Under the assumption that dark energy fills all space, the dark energy of a sphere is identical to the product of its volume and pressure:

$$E_\Lambda = \frac{4}{3}\,\pi\,R^3\,\frac{\Lambda}{\kappa} \tag{18}$$

By using (17) and defining κ as $\frac{8\,\pi\,G}{c^4}$, one obtains the following:

$$\Lambda = \frac{6\,G\,(df - 1)\,M}{c^2\,R^3} \tag{19}$$

Now, we again utilize (9). This leads to the following equation:

$$\Lambda = \frac{df - 1}{df}\,\frac{3}{R_H^2} \tag{20}$$

The factor $\frac{df-1}{df}$ represents the time-dependent ratio of the dark energy to the total energy. According to the Λ-CDM model, this factor is equal to approximately 0.68 at present and will approach a value of 1 over time. Because $\frac{df-1}{df}$ is identical to Ω_Λ, we write the following:

$$\Lambda = \Omega_\Lambda\,\frac{3}{R_H^2} \tag{21}$$

This equation is equivalent to the following equation for Λ derived from the first Friedmann equation, given in (4):

$$\Lambda = \frac{3}{c^2}\,(H^2 - \frac{8\,\pi\,G\,\rho_M}{3}) \tag{22}$$

With $\rho_M = \Omega_M\,\frac{3\,H^2}{8\,\pi\,G}$, $\Omega_M = 1 - \Omega_\Lambda$ and $H = \frac{c}{R_H}$, one can derive (21) from (22). However, only in (21) is Λ defined in terms of fundamental cosmological magnitudes on the basis of the standard assumptions.

5 Conclusion

The G-equation is a provable equation within the framework of modern cosmology. Furthermore, the G-equation is empirically testable and falsifiable. In addition to meeting the criteria of theory-dependent provability and falsifiability, the G-equation also enhances the explanatory power of standard cosmology because of the following features:

F1: Elimination of G as a free parameter.
F2: Implementation of a principle within the mathematical framework of a theory, namely, Dicke's form of Mach's principle within modern cosmology.
F3: Manifestation of the connections among basic physical magnitudes within the equations, in this case, by replacing $\frac{G}{c^2}$ with $(\frac{R}{R_H})^2 \frac{1}{2df} \frac{R}{M}$ in prominent equations of cosmology.

By virtue of these features, the G-equation increases the explanatory power of modern cosmology in the sense that it allows one to identify the causally relevant and, in this sense, fundamental factors directly from the equations. In referring to the "explanatory power of a theory", we do not intend to refer to any standard definition of this term, nor do we wish to engage in a discussion of explanation. Instead, we wish to illustrate a particular understanding of explanatory power that is related to the identification of causally relevant factors within equations.

However, the G-equation also makes manifest that explaining G on the basis of the standard assumptions implies dark energy. This is a not-yet-understood hypothetical entity. The need to refer to such an entity may be regarded as too high a cost for explaining G. It may even be said that a satisfactorily beautiful theory would not allow for such an entity. We have argued neither that modern cosmology is true nor that it is beautiful. Instead, we have focused on deriving equations from this framework that reveal its philosophically significant implications. One cannot demand that a theory be beautiful, much as one cannot demand a beautiful universe. However, one can demand equations that reveal the fundamental features of a theory. This demand is satisfied by the G-equation within the framework of modern cosmology.

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